Formulae of Curvature (K) and Torsion (τ) of Curve:

Let $r = r(t)$ be eqn of curve $\dot{r}, \ddot{r}, \dddot{r}$

then $$K = \frac{|\dot{r} \times \ddot{r}|}{|\dot{r}|^3}, \quad \tau = \frac{[\dot{r}, \ddot{r}, \dddot{r}]}{|\dot{r} \times \ddot{r}|^2}$$

Ex: Prove that $[r', r'', r'''] = K^2 \tau$

Solve - Now $r' = t$, So that $r'' = t' = Kn$

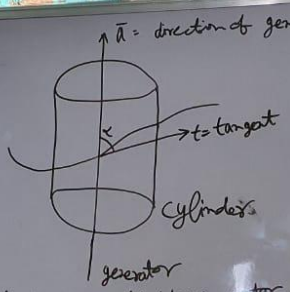
And $r''' = K'n - K^2 t + K\tau b$

Hence $[r', r'', r'''] = r' \cdot [r'' \times r''']$

$= t \cdot [Kn \times (-K^2 t + K'n + K\tau b)]$

$= t \cdot [K^3 b + K^2 \tau t] \Rightarrow K^2 = \tau$

Cylindrical Helices:



A curve which lies on cylinder and cuts the generator at constant angle is called cylindrical helix.

Let $(\vec{a})$ be the unit direction of generator and t is the unit tangent vector to curve.

Then angle between two vector is $\vec{t} \cdot \vec{a} = |t||a| \cos\alpha$

$= 1 \times 1 \times \cos\alpha$

This is the eqn of cylindrical helix. $\vec{t} \cdot \vec{a} = \cos\alpha$ ——— (1)